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Jacobi's Nome (q) in Astronomical Formulæ with Numerical Tables. By R. T. A. Innes.

The Nome q, introduced by Jacobi, can be used in several astronomical formulæ, chiefly as a convenient means of calculating the values of the complete elliptic integrals. It is defined by the following equations:—

$$q = \varepsilon^{-\pi\frac{\mathrm{K}_1}{\mathrm{K}}}$$

wherein

$$\mathrm{K} = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\chi}{\sqrt{1-k^2\sin^2\chi}}\,; \qquad \mathrm{K}_1 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\chi}{\sqrt{1-k_1^2\sin^2\chi}}$$

and

$$k^2 + k_1^2 = 1.$$

We also have

$$\mathrm{K} = 1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot3}{2\cdot4}\right)^2 k^4 + \left(\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\right)^2 k^6 + \&c.$$
$$= \left[1 + 2q + 2q^4 + 2q^9 + 2q^{16} + \&c.\right]^2$$

From these formulæ we may derive

$$q = \tfrac{1}{16}k^2 + \tfrac{1}{32}k^4 + \tfrac{21}{1024}k^6 + \tfrac{31}{2048}k^8 + \&c. \quad \ldots \quad \ldots \quad (a)$$
$$q^{\frac{1}{4}} = \tfrac{1}{2}\sqrt{k} + 2(\tfrac{1}{2}\sqrt{k})^5 + 15(\tfrac{1}{2}\sqrt{k})^9 + 150(\tfrac{1}{2}\sqrt{k})^{13} + \&c. \quad \ldots \quad (b)$$

Schwarz ("Formeln und Lehrsätze zum Gebrauche der elliptischen Functionen," Art. 40) remarks that the nome q and every power of that quantity of which the exponent is a positive number, may be represented by series proceeding according to powers of k, the coefficients of the different terms of these series are positive numbers, and their sum in each case is equal to unity. Unless k is very small the convergency of the two series (a) and (b) leaves something to be desired.

For practical work the series:—

$$q = \tfrac{1}{2}l + 2(\tfrac{1}{2}l)^5 + 15(\tfrac{1}{2}l)^9 + 150(\tfrac{1}{2}l)^{13} + \&c.$$

where

$$l = \frac{1 - \sqrt{k_1}}{1 + \sqrt{k_1}},$$

or if $k = \sin\theta$

$$l = \frac{1}{2}\frac{\sin^2\frac{\theta}{2}}{(1 + \sqrt{\cos\theta})^2}$$

is very convergent.

As K_1 bears the same relation to k_1 that K does to k, it is

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evident that if q is tabulated as a function of θ, it will not be necessary to extend the table beyond 45°.

Let q_1 stand to k_1 as q does to k, then

$$q = \epsilon^{-\pi\frac{K_1}{K}} \qquad \text{and} \qquad q_1 = \epsilon^{-\pi\frac{K}{K_1}}$$

and

$$\log\frac{1}{q} = \pi\frac{K_1}{K}\log\epsilon\,; \qquad \log\frac{1}{q_1} = \pi\frac{K}{K_1}\log\epsilon$$

whence

$$\log\frac{1}{q}\,.\,\log\frac{1}{q_1} = (\pi\log\epsilon)^2$$

or by taking logarithms

$$\text{log of log}\,\frac{1}{q} + \text{log of log}\,\frac{1}{q_1} = 0{\cdot}269868368 \;\ldots \qquad (c)$$

Several tables of q, $\log q$ and log of log $\frac{1}{q}$ have been computed, viz. :—

1840.	Jacobi.	Log q to 5 places from 0° to 90° at intervals of 0°·1. (*Crelle's Journal,* vol. xxvi. p. 93.)
1841.	Loxhay.	Log of log $\frac{1}{q}$ to 12 or 14 places from 0° to 45° at intervals of 0°·1. (In Verhulst's *Fonctions Elliptiques*, Bruxelles.)
1860.	Meissel.	Log q to 8 places from 0° to 90° at intervals of 0°·1. (*Sammlung mathematischer Tafeln* 1, Iserlohn.)
1870.	Bertrand.	Log q to 5 places from 0° to 90° at intervals of 5′, and of log of log $\frac{1}{q}$. (*Calcul Intégral*, Paris.)
1877.	Glaisher.	Of q and $\log q$ to 8 places for every degree from 0° to 89°. (*Monthly Notices, R.A.S.*, vol. xxxvii. pp. 372–4.)
1879.	Schömlich.	Log q to 5 places for every degree. (*Vorlesungen der höheren Analysis*, p. 448.)
1898.	Levy.	Log q to 5 places from Bertrand *C.I.* (*Fonctions Elliptiques*, Paris.)
1901.	Hoüel.	Log q to 4 places for every centesimal degree. (*Tables Numériques*, Paris.)

(See also Glaisher, Tables, *Ency. Brit.*)

The defect of those tables of the above which have sufficient decimals * for astronomical purposes is their lack of differences, so that on each occasion the tables are used the interpolation would be a longer work than a computation of the function from one of the above formulæ.

I had therefore commenced a table to every minute of the first 45°, giving q to ten places of decimals, with $\log q$ and log of log $\frac{1}{q}$

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* I have not seen Meissel's Table.

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to eight places. This work was not persevered in for several reasons. For tables of not very frequent use they would occupy considerable space. Interpolations would still require the use of second differences in the most favourable part of the table (near 45°) for log q, whilst for small angles interpolation fails altogether in the cases of log q and log of log $\frac{1}{q}$. The slow rate of change of a function

$$\frac{q}{\tan^2\frac{\theta}{2}}$$

suggested that the tabulation of that quantity would suit all ordinary requirements.

Short specimens of the earlier computed tables are given :—

θ		q	Δ	log q.	Δ	log of log $\frac{1}{q}$.	Δ
°	′						
0	0	0·000,0000,000+	53	$\overline{\infty}$		∞	
	1	053	159	$\bar{9}$·7233,3226	Too large to be tabulated.	0 9178,5552	Too large to be tabulated.
	2	212	264	$\bar{8}$·3253,9227		·8850,5619	
	3	476	370	·6775.7482		·8646,5494	
	4	846	476	$\bar{8}$ 9274.5233		·8495,7588	
	5	1,322	582	$\bar{7}$·1212,7241		·8375,0811	
	6	1,904	687	·2796,3497		·8273,9286	
	7	2,591	794	·4135,2863		·8186,5281	
	8	3,385	899	·5295,1262		·8109.3699	
	9	4,284+	1005	·6318,1777		·8040,1548	
	10	5,289		$\bar{7}$·7233.3286		·7977,2910	
.	.	.	.	.	.	.	.
°	′						
44	50	0·042,8540,371	358960	8·6319,9174	36363	0·1360,8872	11545
	51	8899,331	359163	23,5537	353	59,7327	5
	52	9258,494	359369	27,1890	343	58,5781	5
	53	9617,863	359573	30,8233	334	57,4236	6
	54	9977,436	359779	34,4567	324	56,2690	5
	55	3,0337,215	359984	38,0981	314	55,1145	5
	56	0697,198	360188	41,7205	304	53,9600	6
	57	1057,386	360394	45,3509	295	52,8054	5
	58	1417,780	360598	48,9804	285	51,6509	5
	59	1778,378	360805	52,6089	36276	50,4964	11546
43	0	0·043,2139,183		8·6356,2365		0·1349,3418	

The table of log $\frac{q}{\tan^2\frac{\theta}{2}}$ which follows was therefore computed.

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Vega's 10-place tables (Italian edition) were used, hence the tenth decimal is somewhat doubtful. The calculations were checked by differencing. The higher differences in several parts of the table suggested alterations of a few units in the last decimal place of several of the functions ; these alterations were adopted, so that finally the run of the differences was smooth enough. Four orders of differences were then changed into differential coefficients and divided by 1, 2, 6 and 24 respectively. The logarithms of the resulting numbers are given. This makes the use of the table in actual practice exceedingly easy, as simple powers of the decimal portion of any given argument are only wanted. I could not derive a workable formula showing the literal forms of the differential coefficients, so that the numbers given probably lack rigid accuracy. Numerous tests, taking the argument positively and negatively (*e.g.* 23° + 6′ and 24° — 54′) seldom differed by more than one or two units in the last decimal place. Even so q will be furnished correctly to at least 11 places of decimals, an accuracy more than sufficient for any practical requirements.

Log q was computed from the working formula

$$\log q = \log \frac{l}{2} + 2\mathrm{M}q^4 - 3\mathrm{M}q^8 \qquad (\mathrm{M} = 0{\cdot}434\ldots)$$

In this formula the approximate value of q required was taken from Glaisher's table already mentioned. The term $3\mathrm{M}q^8$ amounts to one unit in the 11th place of decimals of log q at 45°. The total correction to the approximate formula

$$\log q = \log \frac{l}{2}$$

is shown in a table at the end of this paper. It is arranged so that to seven places of decimals no interpolation is necessary.

Auxiliary Table for the Computation of log q.

$$f(\theta) = \log \left[\frac{q}{\left(\left(\tan \frac{\theta}{2} \right) \right)^2} \right]$$

θ	$f(\theta)$.	$\log f^{\mathrm{I}}(\theta)$.	$\log \frac{1}{2} f^{\mathrm{II}}(\theta)$.	$\log \frac{1}{6} f^{\mathrm{III}}(\theta)$.	$\log \frac{1}{24} f^{\mathrm{IV}}(\theta)$.
0°	9·3979400,087	— ∞	— ∞	— ∞	0·77
1	00,093	1·3979	1·5798	1·3979	·78
2	00,187	2·3032	2·1790	1·7076	·79
3	00,597	2·8325	2·5328	1·8808	·80
4	01,701	3·2087	2·7832	2·0004	·81
5	04,033	3·4998	2·9777	2·1038	·82
6	08,278	3·7380	3·1374	2·1875	·83
7	15,281	3·9397	3·2728	2·2577	·84
8	26,046	4·1147	3·3906	2·3181	·85
9	9·3979441,741	4·2693	3·4947	2·3729	0·86

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θ	$f(\theta)$.	$\log f^{\mathrm{I}}(\theta)$.	$\log \frac{1}{2} f^{\mathrm{II}}(\theta)$.	$\log \frac{1}{\lfloor 3} f^{\mathrm{III}}(\theta)$.	$\log \frac{1}{\lfloor 4} f^{\mathrm{IV}}(\theta)$.
10°	9·3979463,698	4·40782	3·5884	2·4232	0·87
11	79493,421	4·53340	3·6735	2·4698	·88
12	79532,589	4·64831	3·7516	2·5119	·90
13	79583,061	4·75426	3·8239	2·5527	·91
14	79646,882	4·85264	3·89137	2·5911	·92
15	79726,293	4·944492	3·95453	2·6274	·93
16	79823,733	5·030677	4·01406	2·6618	·95
17	79941,848	5·111897	4·07041	2·6955	0·97
18	80083,503	5·188749	4·12405	2·7284	1·00
19	9·3980251,790	5·261719	4·17522	2·7597	1·02
20	9·3980450,038	5·331227	4·22425	2·7903	1·04
21	0681,826	5·397617	4·27131	2·8209	·06
22	0950,991	5·461199	4·31666	2·8506	·08
23	1261,647	5·522241	4·36048	2·8797	·10
24	1618,196	5·580968	4·40290	2·9090	·12
25	2025,345	5·637588	4·44406	2·9375	·15
26	2488,124	5·692279	4·48411	2·9661	·18
27	3011,906	5·7452029	4·52313	2·9943	·20
28	3602,426	5·7965022	4·56126	3·0224	·23
29	9·3984265,805	5·8463056	4·59852	3·0504	1·26
30	9·3985008,569	5·8947253	4·63502	3·0785	1·28
31	85837,679	5·9418686	4·67087	3·1065	·31
32	86760,563	5·9878312	4·70609	3·1345	·34
33	87785,144	6·0326993	4·74076	3·1626	·38
34	88919,872	6·0765523	4·77493	3·1909	·40
35	90173,764	6·1194634	4·80866	3·2193	·43
36	91556,443	6·1615007	4·84201	3·2473	·46
37	93078,189	6·2027275	4·87501	3·2765	·48
38	94749,979	6·2432005	4·90770	3·3056	·53
39	9·3996583,544	6·2829758	4·94013	3·3349	1·56
40	9·3998591,427	6·3221033	4·972342	3·3642	1·59
41	9·4000787,048	6·3606305	5·004360	3·3939	·63
42	3184,773	6·3986027	5·036233	3·4241	·66
43	5799,995	6·4360634	5·068004	3·4547	·70
44	8649,221	6·4730529	5·099701	3·4856	·74
45	9·4011750,169	6·5096105	5·131346	3·5169	1·79

$$\text{log of log } \frac{1}{q^{\mathrm{I}}} = 0{\cdot}2698683{,}680 - \text{log of log } \frac{1}{q}.$$

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To find q we have

$$\log q = \log \tan^2 \frac{\theta + h}{2} + f(\theta) + h f^{\text{I}}(\theta) + h^2 \tfrac{1}{2} f^{\text{II}}(\theta) + h^3 \tfrac{1}{\underline{|3}} f^{\text{III}}(\theta)$$

$$+ h^4 \tfrac{1}{\underline{|4}} f^{\text{IV}}(\theta).$$

As an example let us compute $\log q$ for $\theta = 44^\circ\ 4'\ 18''{\cdot}31$ (or $44^\circ + {\cdot}071753$ or $45^\circ - {\cdot}928247$).

Table 44° =		9·400,8649,221.	45° =	9·401,1750,169
1st diff.	= +	213,252.	= −	3001,055
2nd „	= +	648.	= +	116,593
3rd „	= +	1.	= −	2,630
4th „	= +	0.	= +	46
		9·400,8863,122.		9·400,8863,123

and $\log \tan^2 \frac{\theta}{2} = 9{\cdot}214{,}3839$ gives $\log q$ 8·615,2702.

A recent interesting application of the nome to astronomical formulæ has been made by Professor G. W. Hill in the secular perturbations of the planets (*Amer. Journ. Mathem.* vol. xxiii. No. 4, and *Astron. Journ.* No. 511). Professor Hill shows that the secular perturbations depend on two quantities, $\mathfrak{K}$ and $\mathfrak{L}$, which have the following values in the notation of this paper :

$$\left.\begin{aligned} \mathfrak{K} &= \frac{\text{K}}{k^2_1} = \frac{1}{k^2_1}(1 + 2q + 2q^4 + \ \&\text{c.})^2 \\ \mathfrak{L} &= \frac{\text{K} - \text{E}}{k^2 k^2_1} = \frac{8\,(q - 4q^4 + 9q^9 - \&\text{c.})}{(\text{K}k_1)^{\frac{3}{2}} k^2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \text{(A)}$$

In these formulæ, if θ ($= \sin^{-1} k$) is under 45°, q^9 will not affect the 11th decimal, and may be neglected without appreciable error.

When θ exceeds 45°, q increases in value, and its 9th power may become sensible. If so, it may be better to avoid going beyond 45° by the following device.

From

$$q = \epsilon^{-\pi\frac{\text{K}_1}{\text{K}}}$$

we have

$$\text{K}_1 = \text{K}\,\frac{\log \frac{1}{q}}{\pi \log \epsilon} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(B)}$$

and from

$$\text{E}_1\text{K} + \text{E}\text{K}_1 - \text{K}\text{K}_1 = \frac{2}{\pi}$$

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we have

$$K_1 - E_1 = \frac{1}{K}\left(EK_1 - \frac{2}{\pi}\right)$$

$$= \frac{1}{K}\left(EK\frac{\log\frac{1}{q}}{\pi\log\epsilon} - \frac{2}{\pi}\right) \quad \ldots \quad \ldots \quad (B_1)$$

where

$$EK = \frac{1 + 3^2q^2 + 5^2q^6 + \&c.}{1 + q^2 + q^6 + \&c.}$$

$$\left(\log\frac{1}{\pi\log\epsilon} = 9{\cdot}86506582,\ \frac{2}{\pi} = 0{\cdot}63661977\right).$$

To apply the above formulæ to the example given by Professor Hill (*Astron. Journ.* No. 511), we have $\theta = 45^\circ\ 55'\ 41''{\cdot}69$ or $90^\circ - 44^\circ\ 4'\ 18''{\cdot}31$. Log q has already been computed for the latter angle ($44^\circ\ 4'\ 18''{\cdot}31$) and by equation ($c$), or by the formula at the bottom of the table, p. 498, we get log q for $\theta = 8{\cdot}6556778$.

In this case to compute $\mathfrak{K}$ and $\mathfrak{L}$ it would obviously be shorter to use the set of formulæ (A), but to exhibit the use of the alternative formulæ (B and B_1) they are employed.

$$\begin{array}{rlcrl} & & & \log \text{ of } \log\frac{1}{q} & = 0{\cdot}1413650 \\ 1+2q & = 1{\cdot}082470806 & & \log\frac{1}{\pi\log\epsilon} & = 9{\cdot}8650658 \\ 2q^4 & = \phantom{1{\cdot}08247}5782 & & & \\ \hline \sqrt{K} & = 1{\cdot}082476588 & & & 0{\cdot}0064308 = (a) \end{array}$$

hence

$$\begin{array}{ll} \log K = & 0{\cdot}0688370 \\ 1/k^2 & 0{\cdot}3153325 \\ (a) & 0{\cdot}0064308 \\ \hline \log \mathfrak{K} & 0{\cdot}3906003 \end{array}$$

Then

$$EK = \frac{1{\cdot}015303347}{1{\cdot}001700363}$$

$$\frac{EK\log\frac{1}{q}}{\pi\log\epsilon} = 1{\cdot}0287003$$

and finally

$$\log\mathfrak{L} = 0{\cdot}1270542$$

It should be noted that Professor Hill's expression for $\mathfrak{L}$ as given in the *Astron. Journ.* is approximate; but as the error is of the order q^9 it is generally negligible.

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Several applications of the nome occur in Gyldén's works; one of these is the subject of Dr. Glaisher's paper in the *Monthly Notices.*

As is well known, the elliptic integrals can be used in computing the $b^{(i)}_{\frac{1}{2}}$ coefficients in the development of the perturbative function, thus :

$$b^{(0)}_{\frac{1}{2}} = 2\mathrm{K}. \qquad\qquad (a = k)$$

$$b^{(1)}_{\frac{1}{2}} = \frac{2(\mathrm{K}-\mathrm{E})}{k} = \frac{16(q - 4q^4 + 9q^9 - \&c.)}{\mathrm{K}^{\frac{3}{2}} k\, k_1}$$

$$a\frac{db^{(0)}_{\frac{1}{2}}}{da} = \frac{2(\mathrm{E} - k_1{}^2\mathrm{K}}{k_1{}^2} = \frac{16(q + 4q^4 + 9q^9 + \&c.)}{\mathrm{K}^{\frac{3}{2}} k_1{}^2}$$

With the major planets $\theta = \sin^{-1}a$ only exceeds 45° in the case of *Venus* and the Earth, and then only slightly ($\theta = 46° \, 19' \, 49''{\cdot}04$ after Le Verrier). For the latter angle, q is 0·04616097, so that q^9 is entirely negligible. If the ratio a gave a very large angle for θ, the formulæ already quoted can be adapted so that the computation may be made with a value of q for $90° - \theta$. It is somewhat doubtful if the use of the nome in this connection, except in a few special cases, would be found so useful as the Bruns-Harzer method of mechanical quadratures. (See *Die Säcularen Veränderungen der Bahnen der grossen Planeten*, by Paul Harzer, p. 60, also a paper by the same author in the *Mém. de l'Acad. Imp. de St.-Pétersbourg*, 1886, p. 28).

Applications to Geodetical Astronomy will be found in Halphen, *Fonctions Elliptiques*, vol. ii., and in Hoüel, *Tables Numériques.*

Incidentally the following table of log q and log of log $\frac{1}{q}$ each to 10 places was prepared. Log q is new, but log of log $\frac{1}{q}$ has been taken from Loxhay's table; it is therefore possible that there is a want of rigid correspondence between the two sets of numbers, as the values of log q have not been corrected in their last decimals by differencing.

Table of log q and log of log $\frac{1}{q}$.

θ.	Log q.	Log of log $\frac{1}{q}$.	θ.	Log q.	Log of log $\frac{1}{q}$.
0°	$-\infty$		5°	6·6781266244	5213830738
1	5·2796568024	0·6739735756	6	6·8367323424	5001359410
2	5·8817829559	6147092317	7	6·9709136917	4813116479
3	6·2340757853	5758715761	8	7·0872300618	4643061837
4	6·4841077801	5460355533	9	7·1899124762	0·4487198467

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θ.	Log q.	Log of log $\frac{1}{q}$.
10°	7·2818499344	0·4342734299
11	7·3651031297	4207636216
12	7·4411936923	4080374126
13	7·5112772567	3959765166
14	7·5762522110	3844874259
15	7·6368308344	3734948114
16	7·6935874304	3629369962
17	7·7469918430	3527627640
18	7·7974333790	3429290534
19	7·8452382508	0·3333992574
20	7·8906825461	0·3241419463
21	7·9340020238	3151298923
22	7·9753996364	3063393105
23	8·0150513698	0·2977492718
24	8·0531108519	2893412243
25	8·0897130092	2810986181
26	8·1249770129	2730065964
27	8·1590086662	2650517442
28°	8·1919023594	2572218795
29	8·2237426822	0·2495058801
30	8·2546057618	0·2418935379
31	8·2845603835	2343754355
32	8·3136689325	2269428411
33	8·3419881948	2195876184
34	8·3695700358	2123021482
35	8·3964619813	2050792612
36	8·4227077213	1979121773
37	8·4483475420	1907944537
38	8·4734187025	1837199374
39	8·4979557644	0·1766827230
40	8·5219908792	0·1696771141
41	8·5455540438	1626975885
42	8·5686733251	1557387650
43	8·5913750603	1487953733
44	8·6136840351	1418622246
45	8·6356236462	0·1349341840

Table showing Error of the Assumption that $\log q = \log\frac{l}{2}$.

[In units of the 7th place of decimals.]

θ.	$\log q = \log\frac{l}{2} +$.
0 to 11°	0·000
12	·001
13	·001
14	·002
15	·003
16	·005
17	·008
18	·013
19	·021
20	·032
21	·047
22	·069
23	·100
24	·142
25	·199
26	·275
27	·376
28	·509
29	·682
30	0·906
31°	1·194
31·5	1·366
32	1·561
32·5	1·781
33	2·027
33·5	2·306
34	2·613
34·5	2·960
35	3·347
35·5	3·780
36	4·262
36·5	4·800
37	5·398
37·5	6·063
38	6·800
38·5	7·618
39	8·524
39·5	9·527
40	10·636

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θ.	log $q=\log\frac{l}{2}+$.	θ.	log $q=\log\frac{l}{2}+$.
40° 0′	10·636	42° 40	18·802
10	11·031	50	19·467
20	11·439	43 0	20·152
30	11·961	10	20·859
40	12·297	20	21·588
50	12·748	30	22·341
41 0	13·214	40	23·119
10	13·695	50	23·921
20	14·192	44 0	24·749
30	14·705	10	25·602
40	15·235	20	26·483
50	15·784	30	27·391
42 0	16·350	40	28·328
10	16·934	50	29·294
20	17·537	45 0	30·291
30	18·159		

Reductions of Extra Meridian Observations of Planets.

By P. H. Cowell.

(*Communicated by the Astronomer Royal.*)

The following method of reducing observations of planets with the altazimuth has been brought into use at the Royal Observatory, Greenwich.

From the observation of, let us suppose, the Moon, after correcting for instrumental errors and refraction, we have the data :

(1) At time t_1 the Moon's limb was on the standard azimuth (*e.g.* prime vertical, 45° from meridian, &c.)

(2) At time t_2 the Moon's limb was at zenith distance z, after allowing for refraction.

The method aims at transforming these data into the following :—

(1) At time T the Moon's centre was on the standard azimuth as seen from the Earth's centre.

(2) At the same time T the Moon's centre, as seen from the centre of the Earth, was at an inclination Z to the direction of gravity at Greenwich.

It is clear that when this transformation has been accomplished with argument Z we can obtain the Moon's geocentric polar distance and hour-angle ; and, combining the latter with T, the right ascension also.